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ABSTRACT

This paper analyzes Brainerd's work in assessing the developmental sequence or ordination and cardination concepts of number, and describes a study which investigated the hypothesis that task-specific difficulty could explain Brainers's data. Three new tasks were designed for the assessment of ordination and cardination and administered to a sample of 42 children from preschool, kindergarten, and first grade classes. Each child was given a total of six different types of tasks; one counting task, two strictly ordinal tasks, two strictly cardinal tasks, and one task with both cardinal and ordinal aspects. Results indicated that (1) the Brainerd cardination task was significantly more difficult than any other ordination or cardination tasks; (2) generally, no other tasks were significantly different from one another, and (3) some a posteriori analyses were in the opposite direction predicted by an ordinal theory of number. These results were interpreted as supporting the task-specific difficulty hypothesis. (Author/GO)

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Cardination and Ordination Learning in Young Children

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Running head: Cardination and Ordination Learning

Abstract

An analysis of the tasks used by Brainerd (1973a) to assess the developmental sequence of ordination and cardination concepts of number led to a possible alternative hypothesis-task specific difficulty. Three new tasks were developed to test this hypothesis. Results from middle-class pre-school, kindergarten, and first-grade children showed, (1) the Brainerd cardination task was significantly more difficult than any other ordination or cardination tasks, (2) generally, no other tasks were significantly different from one another, and (3) some a posteriori analyses were in the opposite direction predicted by an ordinal theory of number. These results were interpreted as supporting the task-specific difficulty hypothesis.

Cardination and Ordination Learning in Young Children

Recent work (Brainerd, 1973 a & b, Brainerd & Fraser, 1975) has helped focus attention on two important points in the investigation of children's acquisition of number. First, empirical investigation of the acquisition of number should be preceded by a logical exposition of the meaning of various mathematical concepts. Second, by proceeding from a mathematical-logical definition of terms to a strict operational definition, investigators avoid a major source of confusion in describing what it is the child knows or does not know. Possibly the whole area of conservation of number (e.g. work by Mehler & Bever, 1967, Bever, Mehler, & Epstein, 1968) has less to do with an investigation of what number concepts children learn, and how and when they learn them, than it does with the acquisition of a much more global conceptual system. Looking to such investigations to answer questions about the acquisition of more specifically-defined number tasks has proved less than satisfactory.

Brainerd's (1973 a & b) investigations centered on ordination, linking quantity to transitive asymetrical relationships among at least three terms, and on cardination,

linking quantity with one-to-one correspondence of elements of pairs of classes. While Brainerd's (1973 a & b) tasks were consonant with mathematical definitions of these same concepts, certain problems arise when his tasks are probed in detail. These problems cast doubt on interpreting his findings as unequivocally supporting an ordinal theory of number development. First, the length and weight ordination tasks were designed using materials perceptually accessible to the child, colored sticks and lumps of clay; while the double row cardination tasks were constructed of relatively remote materials, colored dots laminated in plastic. Second, every double row cardination task, whether constructed to test one-to-one, one-to-many, or many-to-one correspondence, had length of row as a conflicting cue. Neither the length nor weight ordination tasks had any conflicting cues, quite the reverse, these tasks seem sequenced and structured in such a fashion as to insure maximal performance. LaPointe and O'Donnell (1974) have shown that length as a conflicting cue to quantity interfered with correct quantity judgments. Thus, there seemed to be an inhibitory influence present in Brainerd's (1973 a & b) cardination tasks not present in the ordination tasks. The conflicting cues in the cardination task should

somehow have been balanced across all types of tasks. Third, there was a single basic ordination task, comparing two dissimilar objects using an intermediate standard, and a single basic cardination task, comparing two rows of different colored balls varying in number and/or length. Thus, concluding that development of ordinal concepts preceded cardinal concepts in children is premature. An equally plausible hypothesis is that the particular cardination task used by Brainerd is more difficult than the ordination task, independent of any consideration of the quantitative aspects of the tasks.

The following experiment investigated whether Brainerd's data were the result of a true developmental sequence or of the task specific difficulty. Several different ordination and cardination tasks were developed, each consistent with strict mathematical definition. Alternate items for the assessment of ordination were constructed using a seriation type drawing task which called for the subject to produce line segments intermediate in length to neighboring dissimilar line segments. Another ordination task was developed viewing ordination in a slightly different fashion than Brainerd (1973a), though still consistent with the axioms of an ordinal theory of natural numbers. Dantzig

(1933) has suggested that the essence of ordinality was the ability to pass from any number to its successor. On this basis, multiple-choice problems were developed to test children's ability to identify the successor of a number, where number was represented by colored dots within a row. This was attempted using as multiple-choice alternatives the same set, the successor, and the successor's successor to the target set. Finally, a new cardination task was developed by alternating red and blue dots in a single row. This arrangement had the effect of making length a completely irrelevant cue to correct performance and at the same time allowed for testing the child's performance with red and blue dots in either one-to-one or many-to-one correspondence.

Method

Subjects

The subjects were 42 children, 14 in each of three groups. A preschool group from a suburban Arizona preschool contained six girls and eight boys whose ages ranged from 48 to 62 months (mean age 55.8 months). A kindergarten group from a suburban Arizona elementary school contained six girls and eight boys whose ages ranged from 65 to 76 months (mean age 71.6 months). A first grade group from

the same elementary school contained seven girls and seven boys whose ages ranged from 75 to 88 months (mean age 81.5 months). All but three children were white. The schools from which the children were drawn both served middle to upper middle class communities.

Tasks

Each child was given six different types of tasks, one counting task, two strictly ordinal tasks, two strictly cardinal, and one task with both cardinal and aspects.

Counting task. Each child was asked to count as high as he or she could. The experimenter gave up to two prompts and stopped the child after successfully reaching the number 15. Next, the child was given a wire rod with ten movable beads attached and asked to count the beads by touching each bead only once. In both counting procedures the experimenter recorded the highest number successfully reached.

Drawing ordination task. A series of four 20.3 x 27.9 cm laminated display cards was presented to the child. The first card pictured seven vertical green bars (1/3 cm wide) resting on a horizontal black line. The seven bars were graduated from 2.5 to 10.2 cm in height. The remaining

three cards were identical to the first except either the second, fourth or sixth bar was missing. On presentation of the first card the child was told, "Here's a picture of some green sticks." On presentation of each subsequent card the child was told, "Here's another picture of some green sticks, but one is missing. Could you start on the black line and draw in the one that's missing here?" For these cards the experimenter placed a piece of tracing paper with a horizontal black line over the card so that the black lines were collinear. The child drew the missing stick on the tracing paper with a green felt pen the experimenter provided. A response was considered correct if its length was intermediate to the length of its immediate neighbors on the presentation card.

Length ordination task. This task was developed by Brainerd (1973a) and has been described in detail elsewhere. There were two problems in this task.

Cardination task. There were eight single row cardination problems and six double row cardination problems. The single row cardination problems contained a number of pairs of 1.9 cm contiguous, red and blue dots in consistent one-to-one correspondence. Pairs were evenly spaced in a single row across a card. There were three, five, six, or eight

dot pairs. Four of the problems contained three, five, six or eight pairs plus one extra dot. Half of the cards started with blue dots, half with red dots. The three, five and six pair problems were presented on 21.6 x 35.6 cm laminated cards. The eight pair problems were presented on 21.6 x 45.7 cm laminated cards. The child was shown a card and asked, "Are there the same number of red dots as blue dots?" In a separate question the child was asked, "Are there more dots of one color?" If he answered yes, the experimenter asked, "Which color has more dots?" This questioning procedure was developed by Rothenberg (1969) to help identify consistent and inconsistent responders. The double row cardination problems were constructed by Brainerd (1973a) and have been described in detail elsewhere. The questioning procedure used in the single row cardination task was used here.

Mix ordination-cardination task. Each of four problems was presented on a 21.6 x 27.9 cm laminated card. A card contained a row of evenly spaced like colored dots at the top of the card and oriented to the left margin. There were either three, five, six or eight dots in the top row. Below the top row were three additional, identically oriented, rows with evenly spaced same colored dots.

These rows contained either the same, one or two more dots than the original row. The bottom rows were randomly arranged from problem to problem. The child was shown a card and asked (cardination), "Would you pick the row from these bottom rows that has the same number of dots as this top row?" Next the child was asked (ordination), "Would you pick the row down here that has just one more dot than the top row?"

Procedure

Each child was tested individually in a single session by either a white male or female experimenter. Within each group all children received the counting task first. Subsequently, half of the children in each group received the following order of tasks--Mixed, Length Ordination, Drawing, Single and Double Row Cardination. The remaining children received the following order--Single Row Cardination, Double Row Cardination, Mixed, Length Ordination and Drawing. Problems within each of the tasks were randomly arranged. Prior to the Mixed, Single and Double Row Cardination tasks, the children were given instructions prohibiting counting as a strategy for solution of the tasks.

Results

Seven preschool children, and all kindergarten and first-grade children performed without error on the counting

tasks. The performance of the seven preschool counters was compared to that of the "non-counters" on the remaining tasks, but did not reveal any systematic differences. The data for all preschool children were pooled in subsequent analyses.

For the purposes of analysis, each child's proportion of correct responses in seven categories was computed. These categories were: (1) Drawing, (2) Length Ordination, problem one, (3) Length Ordination, problem two, (4) Mixed Cardination responses, (5) Mixed Ordination responses, (6) Single Row Cardination, and (7) Double Row Cardination. Figure 1 shows the mean proportion of correct responses for all groups by category. A 3 x 7 repeated measures analysis of variance was obtained using a variance homogenizing arc

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Insert Figure 1 about here

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sine transformation of proportion of correct responses. There were significant effects of Groups ($F = 23.51$, $df = 2/39$, $p<.001$), Tasks ($F = 13.50$, $df = 6/234$, $p<.001$), and a significant Task x Group interaction ($F = 2.32$, $df = 12/234$, $p<.01$).

An examination of the interaction by means of tests of the simple effects revealed that for each group the differences

between tasks were significant ($p<.01$ for all three groups), and that groups differed significantly for three categories ($p<.01$) but did not differ for the remaining four categories. The categories on which the groups differed were the Mixed Cardination, Mixed Ordination, and Single Row Cardination. A further examination of these significant simple effects by means of a Newman-Keuls technique (with an α level = .05) showed the preschool group performance on the Mixed Ordination and Double Row Cardination was significantly lower than performance on all other tasks except the Single Row Cardination task. For the kindergarten group the Double Row Cardination task was significantly lower than all the other task categories. No other differences were significant. Examining the group differences by the same procedure for each significant task category showed that for the Mixed Cardination task both the kindergarten and first-grade groups were significantly different from the preschool group, but not from each other. For the Mixed Ordination and Single Row Cardination task categories all groups were significantly different from each other.

By using the Rothenberg procedure for separating same and more questions into two distinct questions it was possible to estimate the number of linguistically consistent responders

In all, three tasks, Length Ordination, Single, and Double Row Cardination, had a questioning procedure permitting a Rothenberg analysis. Table 1 presents by individual task the proportion of consistent responses for each group.

Insert Table 1 about here

Table 1 shows children respond consistently most frequently to the Single Row Cardination task, followed by the Length Ordination task, and finally by the Double Row Cardination task.

Dependent sample t-tests ($\alpha = .05$, two-tailed) revealed no differences between problem 1 and problem 2 of the Length Ordination task for any group. Similarly, for each group, there was no difference in performance on Single Row Cardination problems containing an extra dot compared to those not containing an extra dot. For all groups, within the Double Row Cardination task the one-to-one correspondence problems were consistently more difficult than either the one-to-many or the many-to-one problems. Finally, it was found Mixed Ordination performance was significantly lower than Mixed Cardination performance for the kindergarten group but not for either the preschool or first-grade groups.

Discussion

These results did not support the hypothesis that understanding of ordination relations precedes the understanding of cardinal relations. The present study did replicate Brainerd's (1973a) original finding that performance on the Double Row Cardination problems is significantly lower than on the Length Ordination problems. However, the present study found that for preschool children, Mixed Ordination performance was also significantly lower than any of the other tasks except Double Row Cardination, from which it did not differ significantly. Additionally, the Single Row Cardination and Multiple-Choice Cardination task performances were not significantly lower than any ordination task performances in any group. These results seemed more consonant with the hypothesis that the Double Row Cardination task is more difficult than the Length Ordination task for reasons that are independent of purely quantitative considerations. LaPointe and O'Donnell (1974) found length interfered with judgments of numerosity. In the Double Row Cardination task length of the rows always provides conflicting cues to cardinal numerosity. This would not be serious if there were compensatory conflicting cues within the Length Ordination task, but there was none. In the

present study it has been found that other cardination tasks do not differ from the Length Ordination task. The data derived from the Length Ordination, Single and Double Row Cardination tasks using the Rothenberg procedure for identifying consistent responders supported the alternative hypothesis, since the proportion of consistent responses increased from the Double Row Cardination problems to the Length Ordination problems to the Single Row Cardination problems, a result consistent across all groups. Some of the kindergarten and first grade children were asked how they solved the Single Row Cardination task without counting and more often than not they would respond something like, "Because there's two and two and two..." And when asked how they answered correctly to the problem with an extra dot, they would say, "Because there's two and two and ..., and one left over." These seemed precise verbalizations of a cardination strategy.

Consistent with previous findings, many-to-one and one-to-many correspondence were both significantly higher than one-to-one correspondence in the Double Row Cardination task for all three groups. These results were not confirmed by similar comparisons in the Single Row Cardination task.

The tests of simple effects also revealed another interesting finding; for the Length Ordination problems

and the Double Row Cardination tasks the three groups did not differ significantly, though the direction of the differences was generally in the expected direction. There was no readily apparent explanation for this result.

Finally, concerning the Mixed Ordination-Cardination task, children in the kindergarten group performed significantly more poorly on the ordination aspect of the problem than they did on the cardination aspect. The differences in the preschool and first grade groups were consistently, though not significantly, in the same direction. The preschool group's t-value, 2.13, just missed being significant at the .05 level (2.16), while the first grade group seemed to be at ceilings in both aspects of the task. These results were contradictory to what was to be expected from an ordinal theory of number development which would predict better performance on the ordination aspects of the task. Additional investigations are necessary since the present data cast doubt on the generality of previous theorizing about the developmental predominance of ordination concepts. It is especially important to continue and to expand the assessment of ordination-cardination performance within the same task.

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Table 1

Proportion of Consistent Responders for

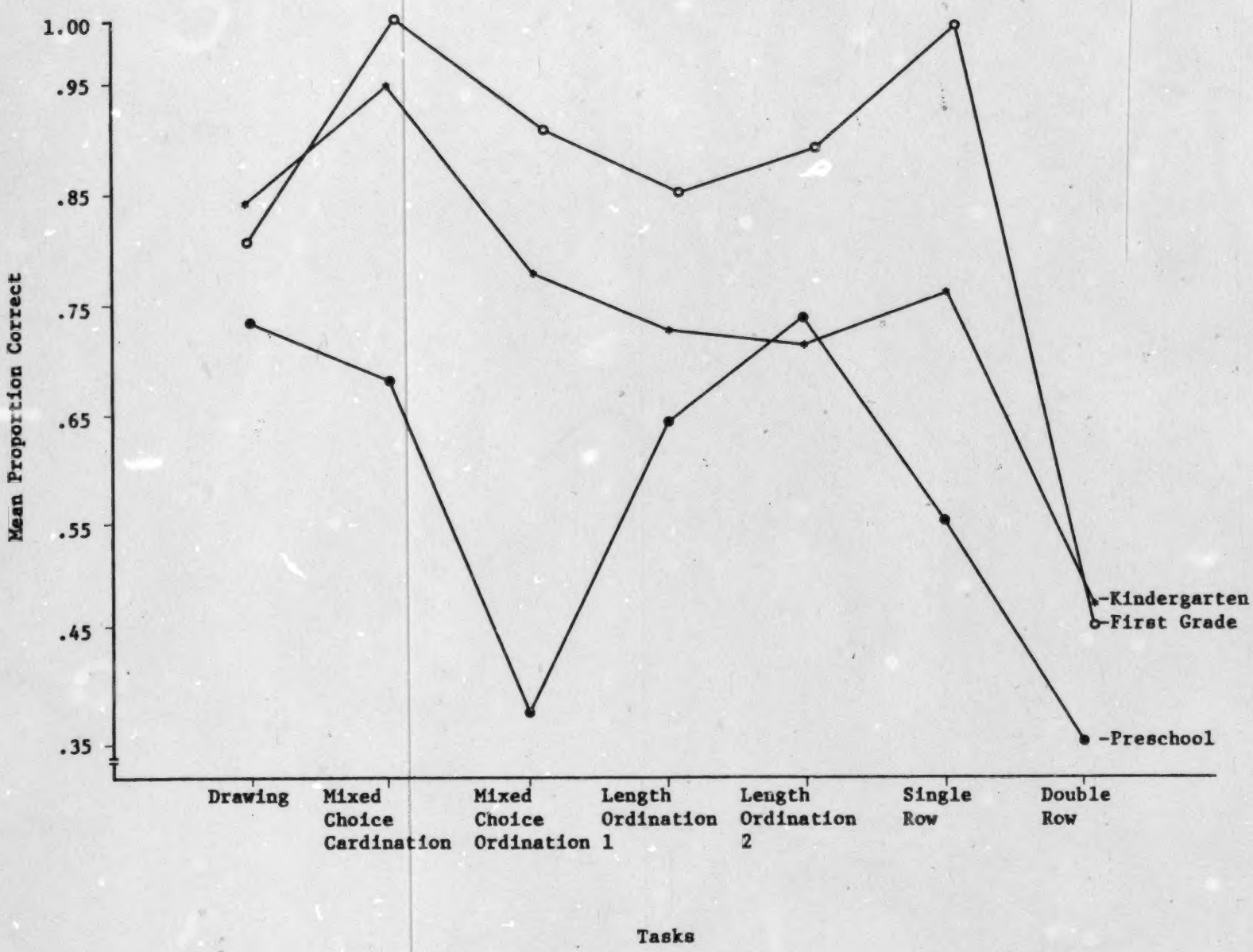
Ordination and Cardination Tasks

	Length Ordination	Single Row[a] Cardination		Double Row[a] Cardination	
		1-1	M-1	1-1	M-1
Preschool (N = 14)	.25	.32	.36	.00	.09
Kindergarten (N = 14)	.43	.66	.71	.07	.36
First Grade (N = 14)	.61	1.00	.93	.18	.46
Total (N = 42)	.43	.66	.67	.08	.30

[a]Tasks are separated into one-to-one and many-to-one categories.

Figure Captions

Figure 1. Mean proportion correct responses for each group for all categories.


1.00
.95
.85
.75
.65
.55
.45
.35
Mean Proportion Correct
-Kindergarten
-First Grade
-Preschool
Drawing
Mixed Choice Cardination
Mixed Choice Ordination
Length Ordination 1
Length Ordination 2
Single Row
Double Row
Tasks